NOTATION

L, length of investigated section of pipe; t, time of passage of liquid over section; v_{av}, mean flow velocity; v, particle velocity; M, magnetization per unit volume of the liquid; I, intensity of NMR signal; f(v), particle velocity distribution function.

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TURBULENT PIPE FLOW OF CONCENTRATED EMULSIONS WITH A NONEQUILIBRIUM DISPERSE PHASE

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Quantitative data are obtained in velocity pulsation damping and the amount by which particle coalescence exceeds the turbulence microscale is calculated for the pipe flow of concentrated emulsions.

The motion of two-phase systems is generally described with the assumption that the laws of conservation of mass, momentum, and energy are satisfied. Meanwhile, the complete system of Navier—Stokes equations should consider the effect of disperse-phase particles on the motion of the dispersion medium, while the equation of motion of the particles should reflect, along with external body forces and forces of interaction between the particles, the effect of the motion of the dispersion medium. As a result, to obtain a closed system of equations, it is necessary to have a kinetic equation describing the dynamic state of the disperse phase, together with the corresponding boundary conditions [1, 2]. Such a system of equations is particularly valuable is analyzing similitude among two-phase flows [3], although it is difficult to obtain final theoretical relations, such as for turbulent flow.

However, the large amount of empirical data available on turbulent flows of unstable emulsions, connected with study of the mixing of mutually insoluble liquids and the rate of phase separation in various commercial processes, can be used to construct a partial semi-phenomenological model of a two-phase system. This will provide us a sufficiently simple basis on which to perform engineering calculations connected with the movement of unstable emulsions. At the same time, the model, being in good agreement with the empirical data, will serve as a reliable basis for semiempirically analyzing the assumptions necessary to analytically study the complete system of equations for a two-phase system.

The goal of the present work is to study anomalous effects and is a consequence of the inadequacy of the homogeneous model of a two-phase system in the case of coarse-dispersed emulsions with a nonequilibrium disperse phase. We also hope to obtain quantitative estimates of the above effects using experimental data.

Inverse Effect of Disperse-Phase Concentration on Turbulent Flow of an Emulsion in a Pipe

It has been shown for emulsions of immiscible liquids of similar densities that the motion of disperse-phase drops in the inertial range of uniform turbulent flow nearly coincides with the motion of the dispersion medium [4]. Also, it is known that an emulsion with a low-viscosity dispersion medium is capable of retaining Newtonian rheological properties up to W = 0.5 in the turbulent regime [5]. In determining effective viscosity in accordance with the Darcy—Weisbach equation for a homogeneous liquid, the Newtonian behavior of the emulsion was controlled by comparing measured and theoretical velocity profiles.

We will show that the turbulence damping in mixers noted in [6] is consisted with the available empirical data for pipes through which unstable emulsions are being pumped in a

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regime which prevents layering of the phases with coalescence of the disperse phase. If it is assumed that the condition $\bar{v}_e(W) \simeq v_{*e}$ [8] holds in the motion of concentrated emulsions for such a quasihomogeneous structure — this condition having been obtained for homogeneous liquids — then we way write the following on the basis of balance of the pressure drop and drag: $\bar{v}_e(W) = \sqrt{\lambda_e(W)/8}U$. Then, considering the uniformity of turbulent flow in a pipe at sufficiently large Reynolds number [4] and using the law $\bar{v}_e(W) = \bar{v}_e/f(W)^{0.5}$ established earlier for mixers [6], we have:

$$\lambda_e(W) = \lambda_e/f(W), \tag{1}$$

where λ_e is determined from the Blasius formula on the basis of the effective viscosity of a finely dispersed emulsion in accordance, for example, with the semiempirical relations [9]:

$$\mu_e = \mu_c \exp[K_R W/(1 - W)]. \tag{2}$$

Equation (1) agrees, to within the numerical coefficient in the expression $f(W) = 1 + \eta(W)$, with the result in [10] obtained for turbulent flow along a 0.05-m-diameter pipe of a concentrated emulsion of transformer oil in water: $\lambda_e(W) = \lambda_e/(1 + 1.125W)$. Thus, despite certain differences in the hydrodynamic structure of a mixer and a pipe, the above should mean that the phenomenon of pulsation damping should also be important in the pipe. Moreover, this effect will become predominant [6] when a coarse-dispersed flow contains stabilizers preventing coalescence. However, in finely dispersed emulsions, it is only due to the process of coalescence in flow along fairly long pipes that drops with $\delta > \lambda_0$, capable of reducing the magnitude of turbulent pulsations, may be formed.

It is interesting to note that, by calculating the Reynolds number from the effective viscosity of a finely dispersed emulsion, the concentration relation in Eq. (1), for example, may be related to the quantity μ_e in the expression for Re_e. Then, without allowing for the effect of pulsation damping, the well-established exponential increase in effective viscosity (2) with an increase in concentration will, at the corresponding drop sizes, be diminished by a factor of $f(W)^4$. As a result, the applicability of Einstein's linear law to concentrated emulsions is a consequence of pulsation damping within the framework of the homogeneous liquid model, rather than a basis for interpreting the physical essence of the process reflected by f(W), in accordance with [11].

We will make use of this fact to quantitatively evaluate the constants of the linear concentration relation for pipe conditions. Figure 1a shows how expression of $f(W)^4$, at certain empirically determined values of η, lessen the increase in effective viscosity.

The results of calculations shown in Fig. 1b indicate that f(W) can be used to actually linearize viscosity curves corresponding to finely dispersed concentrated emulsions. For example, the authors of [5] experimentally established the relation $\mu_e/\mu_c = \exp(2.5W)$ for a system composed of water and thin oil. The effective viscosity remains roughly the same if

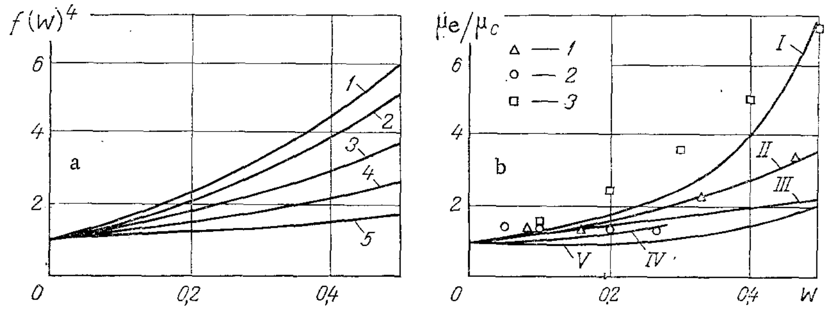

Fig. 1. Evaluation of the effect of turbulence damping in a pipe on the basis of experimental studies of the concentration function f(W) and the effective viscosity μ_e/μ_c: a) 1 — η = 1.125 [10]; 2 — 0.96 [13]; 3 — 0.75; 4 — 0.5; 5 — 0.25; b) theoretical curves: I — Eq. (2), $K_R = 2$; II — exp(2.5W); III — 1 + 2.5W; IV — exp(2.5W)/(1 + 0.5W) ; V — $\exp[2W/(1-W)]/(1 + 0.75W)^4$; experiment: 1, 2 — [5]; 3 — [12].

the disperse phase is replaced by thick oil under the same conditions. Allowing for the fact that an increase in the viscosity of the disperse phase from 0.018 to 0.260 Pa·sec increases the size of the dispersed drops without significantly affecting the character of the viscosity relation, the agreement between curve IV and the experimental point 2 in Fig. 1b reflects the effect of pulsation damping at $\eta = 0.5$. Similarly, with slight deviations, the empirical viscosity data in [12] for a finely dispersed emulsion of carbon tetrachloride in water reduces to Einstein's linear law – III – on the basis of curve 3 in Fig. 1a. In [13], experimental studies of turbulence damping in jet flows of two-phase systems made it possible to establish the value $\eta = 0.96$.

Thus, in contrast to mixers, where $3.14 \leqslant \eta \leqslant 9.0$ [6], turbulence damping in pipes is characterized by $\eta = 0.5$-1.125. The decrease in η is quite satisfactorily explained by a decrease in turbulence intensity from 50-60% in mixers [14] to 3-4% in pipes [4].

Calculation of Coalescence

Drop size changes with the turbulent flow of unstable emulsions in pipes, due to coalescence and comminution of the drops. In the present work, the breakup of coalesced drops is precluded by the condition $\lambda_0 < \delta < d_m$, which is satisfied by the nonequilibrium disperse phase. Based on the material balance of the disperse-phase volume and the number of drops of average size, drop coalescence by means of the pairwise union of colliding droplets is described by a system of two differential equations [15]:

$$\frac{d\delta}{dl} = -\frac{\delta}{3n}\frac{dn}{dl}; \quad \frac{dn}{dl} = -\frac{\theta}{2}\frac{n}{U}. \tag{3}$$

The coalescence process is calculated with allowance for the following assumptions. It is assumed that the turbulent flow is uniform and that the concentrated emulsion in the region of change in W being investigated does not display any anomalous rheological properties. The effect of drop concentration is reflected by the effective viscosity of the emulsion and an additive term with the concentration relation in the formula $\bar{v}_e(W) = \bar{v}_e/f(W)^{0.5}$. In coarse-dispersed emulsions at $\delta_0 > \delta$, the frequency of drop coalescence under the influence of turbulent pulsations, with allowance for the opposite effect of the disperse phase, has the form

$$\theta = K_v \frac{4\sqrt{2}}{\sqrt{3\pi}} \frac{\bar{v}_e W}{f(W)^{0.5}\delta}. \tag{4}$$

We will approximate the average magnitude of the velocity pulsations in a finely dispersed emulsion with the expression $\sqrt{\lambda_e/8}U$. The solution of system (3) relative to the drop diameter, with the boundary condition $\delta|_{t=0} = \delta_0 > \lambda_0$ at W = const, is written as follows:

$$\frac{\delta}{\delta_0} = 1 + K_v \frac{0.4\sqrt{2}}{3\sqrt{3\pi}} \frac{W}{\delta_0 (1+\eta W)^{0.5} \mathrm{Re}_e^{0.125}} l. \tag{5}$$

Figure 2 shows the results of calculation of drop coalescence from 100 to 500 μm under the following conditions: D = 0.2 m, $\mu_c = 0.005$ Pa·sec, $\rho_c = 866$ kg/m³, $\rho_d = 1163$ kg/m³,

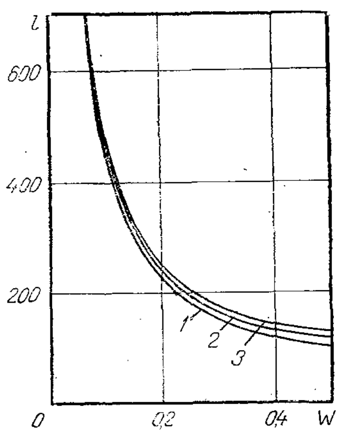

Fig. 2. Dependence of pipe length l (m) on volume concentration of the disperse phase W with a constant degree of drop enlargement $\delta/\delta_0 = 5$, $\delta_0 = 100$ μm; 1-3) $\eta = 0$; 0.5; 1.125.

$\Delta P/\Delta l$ = 100 Pa/m, $K_V = 0.26 \cdot 10^{-4}$, K_R = 1.0. It turned out that the effect of velocity pulsation damping on the coalescence of coarse-dispersed drops is insignificant compared to concentration. An increase in η up to 1.125 increases the length of pipe needed to obtain a specified degree of coalescence by 2.8-13%, depending on the value of W. It is more important to note that flow of an emulsion with a nonequilibrium disperse phase may become unsteady.

Relaxation Model of Flow of Unstable Emulsions

Based on the above analysis of coalescence in concentrated emulsions and the relation adopted for pulsation damping, we may make the following conclusions relative to f(W). When the drop sizes in a turbulent flow of a finely dispersed emulsions are less than the microscopic scale of the turbulence, the numerical coefficient in the linear expression for f(W) is zero, and the effect of the concentration of the disperse phase is completely characterized by the effective viscosity. This effect was studied in [16]. As drop size increases due to coalescence, the sizes will approach the inertial range. In this range, the interaction of the drops with eddies possessing kinetic energy will be of a dynamic nature. In this case, η will increase and reach a maximum value in accordance with the hydrodynamic parameters of the flow and the equilibrium drop size. This is indirectly confirmed by the degeneration of f(W) into a constant at large values of the Weber number [17].

In turn, the drop coalescence which completely determines the rate of increase in η is calculated by means of system (3), substituting the kinetic equation for monodisperse emulsions. In linear approximation (5), f(W) is written thus

$$\eta(\delta) = \eta\left(\frac{\delta - \lambda_0}{d_m - \lambda_0}\right). \qquad (6)$$

Allowing for the variable $\eta(\delta)$ in the expression (4) for drop coalescence frequency and after making simple transformations, the refined solution of system (3), with the boundary condition $\delta|_{l=0} = \lambda_0$, has the form

$$\left[\left(1 + \eta W \frac{\delta - \lambda_0}{d_m - \lambda_0}\right)^{1.5} - 1\right] \frac{(d_m - \lambda_0)\,\mathrm{Re}_e^{0.125}}{0.092 K_v \eta W^2} = l. \qquad (7)$$

Thus, the quasihomogeneous turbulent flow of an emulsion with a nonequilibrium disperse phase, accompanied by drop coalescence, is unsteady and at $\lambda_0 < \delta < d_m$ is relaxational in character. On the basis of the Darcy–Weisbach equation, the pressure drop will change in accordance with the expression

$$\frac{dP}{dl} = \frac{\lambda_e}{1 + \eta(l) W} \frac{\rho_e U^2}{2D}. \qquad (8)$$

Steady-state flow is established after full completion of drop enlargement with $\delta|_{l=l_k} = d_m$, $\eta(l_k) = \eta$ and is subsequently maintained as a result of dynamic equilibrium between the drop comminution and coalescence rates. It was this very regime that was reproduced and studied experimentally in [10], and η = 1.125 corresponded to the maximum equilibrium drop size under the test conditions. Another reason for the appearance of transitional regimes is a shift in dynamic equilibrium in the disperse phase with a change in the cross section of the pipe, the passage of the emulsion through a valve, union, or other local resistance, and the addition of demulsifying agents and stabilizers.

Let us examine the effect of coalescence of the disperse phase on the pressure loss in the pumping of unstable emulsions on the basis of the linear relation between drop size and pipe length obtained in the first approximation (5). After integrating (8) with $\eta(l) = \eta l/l_k$ and the boundary condition $P|_{l=0} = P_0$, we have

$$\frac{P(l) - P_0}{P(l_k)} = \frac{1}{\eta W} \ln\left(1 + \frac{\eta W l}{l_k}\right), \qquad (9)$$

where $P(l_k) = \lambda_e \rho_e U^2 l_k / 2D$. The length of the nonequilibrium section of the pipe is determined with Eq. (7) at $\delta = d_m$, which reduces to Eq. (5) after taking the limit as $\eta \to 0$. A sample calculation for the conditions chosen above at U = 0.88 m/sec gave the following results. With a change in W from 0.1 to 0.5 and fixed $\eta = 0.5$, turbulence damping is characterized by a reduction in the pressure drop by 2.4-10.4%. Under similar conditions, the value $\eta = 1.125$ corresponds to a reduction in the pressure drop relative to the maximum calculated from the effective viscosity of a finely dispersed emulsion from 5.2 to 20.7%.

For rapidly coalescing systems, as $K_v \to 1$ the relaxation processes take place much more rapidly, and the pulsation damping effect is manifest on shorter sections of pipe. Interpretation of the experimental data using the simple homogeneous model leads here to an anomalous reduction in λ_e relative to the Blasius formula. Thus, for a disperse system of low-viscosity liquids, this effect was seen at W = 0.35-0.50 with an increase in the turbulence level [18]. Equations (7) and (9), obtained on the basis of a two-phase model allowing for disperse-phase coalescence and velocity pulsation damping, reflects this effect. The fact that it does is additional evidence of its adequacy.

Thus, the two-phase model makes it possible to unambiguously interpret available experimental data on the pipe flow of unstable emulsions which do not exhibit anomalous rheological properties. The individual physicochemical and hydrodynamic features of actual liquid systems are reflected by the model constants K_v, η, and K_R.

NOTATION

λ_o, microscopic scale of turbulence; W, concentration of disperse phase; U, mean flow rate; $\bar{v}_e$, $\bar{v}_e(W)$, velocity pulsations in finely and coarsely dispersed emulsions, respectively; f(W), concentration function reflecting the effect of turbulence damping; λ_e, effective drag coefficient; K_R, viscosity relation constant; μ_e, μ_c, dynamic viscosity of emulsion and dispersion medium; η, concentration function constant; δ_o, δ, initial and running mean drop sizes; $Re_e = UD\rho_e/\mu_e$ Reynolds number of emulsion; D, pipe diameter; $\rho_e = \rho_c(1 - W) + \rho_d W$, density of emulsion; ρ_c, ρ_d, density of dispersion medium and disperse phase; d_m, maximum drop size stable with respect to comminution; l, pipe length; n, number of drops per unit volume of emulsion; θ, frequency of drop coalescence; K_v, constant of efficiency of drop collision under the influence of turbulent pulsations; P_o, P, pressure at the initial and running points of the pipe; l_k, length of nonequilibrium section of the pipe.

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LAMINAR FLOW OF AN INCOMPRESSIBLE FLUID IN A PLANE CHANNEL WITH UNIFORM UNIDIRECTIONAL INJECTION

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An experimental and theoretical study is made of flow hydrodynamics in a plane channel with a permeable wall.

Problems on gas flow in channels with permeable walls arise in connection with the study of heat and mass transfer processes in heat pipes, in the pore cooling of gas-turbine blades, and in several other important practical applications. The hydrodynamics of a developed flow in a plane channel were examined in [1-5], while flow in the initial section of a channel with symmetrical two-sided injection through the walls was investigated in [6, 7]. The hydrodynamics of a flow in a long narrow channel with one-sided injection were studied in [8]. The present work obtains numerical solutions of motion equations and experimentally studies nonsymmetrical flow in a plane channel with one permeable wall.

The motion equations, describing two-dimensional flow in an approximation of the boundary-layer theory, have the following form in dimensionless variables:

$$\frac{\partial \bar{u}_x}{\partial \bar{x}} + \frac{\partial \bar{u}_y}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u}_x \frac{\partial \bar{u}_x}{\partial \bar{x}} + \bar{u}_y \frac{\partial \bar{u}_x}{\partial \bar{y}} = -\frac{d\bar{p}}{d\bar{x}} + \frac{\partial^2 \bar{u}_x}{\partial \bar{y}^2}. \tag{2}$$

With a uniform velocity profile at the inlet section of the channel, the boundary conditions for Eqs. (1) and (2) will be

$$\bar{x} = 0 \quad \bar{u}_x = 1; \ \bar{y} = 0 \quad \bar{u}_x = 0, \ \bar{u}_y = \mathrm{Re}_V; \quad \bar{y} = 1 \quad \bar{u}_x = \bar{u}_y = 0. \tag{3}$$

Figures 1 and 2 show results of numerical solution of systems (1) and (2), with boundary conditions (3), for different values of the parameter Re_V. Figure 1 shows profiles of the axial velocity component, normalized by the mean velocity in the cross section of interest $U = U_0(1 + \mathrm{Re}_V\bar{x})$, for $\mathrm{Re}_V = 80$. The velocity distribution is clearly S-shaped in the initial section of the channel (curves 2 and 3 in Fig. 1), which is typical of flow in a boundary layer with injection [9]. As $\bar{x}$ increases, the point of inflection of the velocity profile becomes less distinct and the velocity distribution approaches the developed profile found from similarity solution of the motion equations. An increase in injection rate is accompanied by a shift in the velocity maximum toward the impermeable wall, and the distribution of u_x approaches the limiting distribution $u_x = U \sin(\pi\bar{y}/2)$ [5]. Father from the channel inlet, the calculated velocity profiles agree with those measured in [8].

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